Materials Used:
-Graph Paper
-Pencils
-Cardboard boxes, painted with 0-11 (for examples)
-Painter's tape (to set up the polygon on the ground)
-Notecards for the polygon on ground

Introductions:
Hello my name is Danielle Wood, and I am a music major at USC.
Hello my name is Claudio Olivera, and I am an adjunct faculty at USC.

And we are Mathemusik! This presentation is not only a performance, but an interactive experience. You're probably wondering why on earth a couple musicians and a math major are visiting your math class today.

Begin with Leonard Bernstein Quote:

"The fact is that music is not only a metaphorical art[;] it is made of mathematically measurable elements...so any explication of music must combine mathematics with aesthetics."

Performance:
Begin with *Elegie for Horn and Piano* by Frances Poulenc (contains 12-tone row), only play the first few phrases.

Play Mozart's Concerto No. 4 for Horn and Piano (beginning). Kind of a filler piece and helps offset the seriousness of the elegie. (Both pieces are related: the *Elegie* was written in memory of Dennis Brain, and he became famous through playing the Mozart concerti.)

We could talk about Mozart *[Slide.]* who lived in Vienna *[Slide.]* and was composing with a bunch *[Slide.]* of other white guys with terrible taste in hair.

Or, we could look at Poulenc *[Slide.]* and his life in France *[Slide.]*. Which, during this time, still a bunch *[Slide.]* of white dudes were composing music, but at least they dropped the wigs!

[Slide.] For instance, juxtaposing the two pieces you just heard, we can easily note the rhythmic differences, the harmonic changes, the fast versus slow tempo...Though we could focus on comparing and contrasting these two works, we will only discuss on a particular mathematical concept in the Elegie.

First, we're going to hand out some paper and pencils to each of you. All you have to do is write down a few numbers for us. *[slide change]* As you can see on the board, we have a polygon with twelve numbers, 0-11. Take this time now to write down each number in whatever order you'd like, but take care not to repeat any number once you have used it. The board has a few examples for you. *[Slide.]*

[Allow some time for this process to occur. 2-3 min]

Who here is a composer?

Everyone is a composer today. Because those numbers you wrote down are more than just a numerical list; they're musical compositions! Our compositional focus is called a "twelve-tone row." What does that mean? *[next slide]* Each tone is assigned a number, which is called a pitch-class. *[Slide.]* So we will take the a few volunteers' sequences and play them on the piano. *[Claudio plays each tone. Then plays the three sequences.]*

Now we can do the exact same thing with that crazy piece you heard earlier. *[Next slide.]*

As you can see, every note has its corresponding number, which then creates a sequence of twelve numbers. You may wonder why C is "0" versus "1" or why we chose to start with C at all. Why not A? It is the beginning of the alphabet afterall. The piano *[next slide]* has both black and white keys, and in the middle *[click]*, is this note, middle C. *[Claudio plays C on piano.]* Whenever you play the C scale, *[Claudio plays C scale.]* it is comprised of only the white keys, which if you want to get fancy means that there are no sharps or flats, so naturally, we use it as our "0." In mathematical terms, C is our standard basis.

What are some ways we use numbers in our math class?

[Take suggestions; add, subtract, divide, fractions, ratios]

Yes, perfect! Well we can do some of those things with our compositions as well. *[Next slide, depicting row.]* We can transform them and do things like….retrograde. Anyone want to guess what that means?

[Slide.] It simply means to go in reverse order. Start with your last number and work all the way to the first number. So everyone go ahead and try that out on your own sequence.

[Allow 2-3 min for this.]

Awesome! Anyone want to volunteer their original and retrograde works to be played on the piano? *[2-3 min.]*

Now let's look at this row here. I need twelve volunteers for this exercise. *[Create the original row. Each student will receive a box labeled 0-11 and standing side-by-side to recreate the row. After lineup, use rest of class to help move the students into the retrograde. For smaller classes, use 3 students for the example.]* We are going to line up and create this Poulenc row. Now, for the rest of the class, let's help our volunteers move around and create the retrograde!

[5-10 min.]

Great! Let's check our answer with the board! *[Slide.]* Pretty easy, right? I will play what this sounds like for you by first playing the original row, then its retrograde. *[Plays..2-3 min]* Now, let's look at a graph of what we just did. *[Slide.]* Retrograde is similar to having opposite slopes. *[Slide.]* For instance, we started with a high to low relationship, or negative slope, in the original, and the retrograde is the opposite, so a positive slope. *[Point.] [Slide.]* And another. Since this graph is pretty complex, we're going to relate this to a couple graphs we all know and love. *[Slide.]* You begin with y=x. When you "retrograde" the slope, you produce y=-x.

So I think we have retrograde under our belts; how about we check out transpositions or transformations? *[Slide.]* Whenever you transpose, you alter the starting pitch or number while maintaining the relationships between each number or pitch.

[Slide.] To begin with, let's work on the Poulenc row together. So I want to create transformation of 3. What does this mean? Simply adding 3 to each number. *[Slide.]*

So what do we get? 3, 5, and 8... ? *[Slide.]* Yep! Next one - 14, 10, 11? *[Slide.]* Negative! 11 goes to 2 because we start from the beginning again! Think of this polygon as a clock. *[Slide.]* When we return to 12 for the minutes, do we say it's 12:60 pm? Nope! We say 1 pm. Same applies for this situation. Whether you are adding or subtracting, whenever you reach the top, you restart your count. This next one is similar to the second line, will 10 go to 13? Nope! *[Slide.]* And finally, where does the 9 go? *[Slide.]* Now, I will play this for you.

Do you guys want to try it? We're going to focus on the first three numbers of each composition, so everyone go ahead and transform the first three numbers of your row. *[Wait 1-2 min.]* Now I need two compositions and three volunteers. *[Take up compositions. Hand out the respective boxes for the first three numbers of composition one. Explain. Set them up accordingly. Ask students for a transformation (preferably 1-5). Have students move to their spots, keeping their original boxes. Bring out boxes for new coordinates and sit in front of the students. Compare the two. Then sit students down. 5-10 min]* As you may can see, we have created the twelve-sided polygon on the ground! Your fellow classmates are set up in the first volunteered set of pitches *[state the numbers.]*. Now we need a transformation number! Raise your hands to suggest one! Okay guys! Let's move to our new spots.

[Slide.] Similar to the retrograde, we can also represent this concept on a graph. Whenever you move 3, you're simply adding 3 to the graph. For this example, we moved upward. You have the freedom and ability to move up 7 or down 11 or any number in between.

Finally, *[Slide.]* we're going to look at inversions. You maintain the relationships between each number or pitch, similar to transposition. For fancy music talk, *[talk in haughty voice]* you maintain the intervallic relationships, but we're not here for mumbo-jumbo like that. If we focus on the first three pitches of the row, we simply flip the points over the y-axis. *[Slide.]* The original row is comprised of 0, 2, 5. An easy way of doing this is subtracting our number from 12. So our first number is 0. *[Slide.]* We maintain the relationship between our first and second numbers. Since 0 and 2 were two steps apart, we go two steps in the opposite direction *[Slide.]* to get 10. 2 and 5 were three steps apart, so therefore our next number is 7 *[Slide.]*.

Let's check out the graph. *[Slide.]*

The triangle is reflected over the y-axis. Instead of doing this ourselves, I'm going to use this amazing online tool created by one of my theory professors. *[Slide. Click link.]*

We are moving the entire sequence 12 steps. Essentially subtracting the original numbers from 12 will produce our inverted sequence. The cool thing about this program is we don't have to do anything! I simply have to press this "I" key to invert the entire row!

[Input 025. Click "I." Claudio plays this example.]

Let's try out some other compositions from you guys. Can I have a few volunteers? *[Pick 4-5. Examples. Show cluster chord too.]*

Now we're going to listen to a bit of the Poulenc again. This time I want you to really listen in for the things that we've talked about. I also have a challenge for you! There are a few more rows in this piece that we have not gone over. If you can guess how many rows are in this piece, then you will get major props from us! (Maybe some candy.)

[Play piece.]

So how many were there?

[Slide.] There were actually two! One at the beginning, which I played, and one that Claudio played by himself.

[Slide.] Thank you all so so much for letting us come here today! You guys were an awesome audience. Let's give a round of applause to our volunteers! A special thank you to your teacher.

Thanks so much and have a wonderful rest of your day!